Statistical inference: one and two-sample t-tests

Statistical Inference and Science

- Previously: descriptive statistics. “Here are data; what do they say?”.
- May need to take some action based on information in data.
- Or want to generalize beyond data (sample) to larger world (population).
- Science: first guess about how world works.
- Then collect data, by sampling.
- Is guess correct (based on data) for whole world, or not?

Sample data are imperfect

- Sample data never entirely represent what you're observing.
- There is always random error present.
- Thus you can never be entirely certain about your conclusions.
- The Toronto Blue Jays' average home attendance in part of 2015 season was 25,070 (up to May 27 2015, from baseball-reference.com).
- Does that mean the attendance at every game was exactly 25,070? Certainly not. Actual attendance depends on many things, eg.:
  - how well the Jays are playing
  - the opposition
  - day of week
  - weather
  - random chance

Packages for this section

```
library(tidyverse)
```

Reading the attendances

...as a .csv file:

```
my_url <- "http://ritsokiguess.site/datafiles/jays15-home.csv"
jays <- read_csv(my_url)
jays
```

```
# A tibble: 25 x 21
     row  game date   box    team  venue opp   result  runs Oppruns innings wl
   <dbl> <dbl> <chr>  <chr>  <chr> <lgl> <chr> <chr>  <dbl>   <dbl>   <dbl> <chr>
 1    82     7 Monda~ boxs~ TOR   NA    TBR   L          1       2      NA 4-3
 2    83     8 Tuesd~ boxs~ TOR   NA    TBR   L          2       3      NA 4-4
 3    84     9 Wedne~ boxs~ TOR   NA    TBR   W         12       7      NA 5-4
 4    85    10 Thurs~ boxs~ TOR   NA    TBR   L          2       4      NA 5-5
 5    86    11 Frida~ boxs~ TOR   NA    ATL   L          7       8      NA 5-6
 6    87    12 Satur~ boxs~ TOR   NA    ATL   W-wo       6       5      10 6-6
 7    88    13 Sunda~ boxs~ TOR   NA    ATL   L          2       5      NA 6-7
 8    89    14 Tuesd~ boxs~ TOR   NA    BAL   W         13       6      NA 7-7
 9    90    15 Wedne~ boxs~ TOR   NA    BAL   W          4       2      NA 8-7
10    91    16 Thurs~ boxs~ TOR   NA    BAL   W          7       6      NA 9-7
# i 15 more rows
# i 9 more variables: position <dbl>, gb <chr>, winner <chr>, loser <chr>,
#   save <chr>, `game time` <time>, Daynight <chr>, attendance <dbl>,
#   streak <chr>
```

Another way

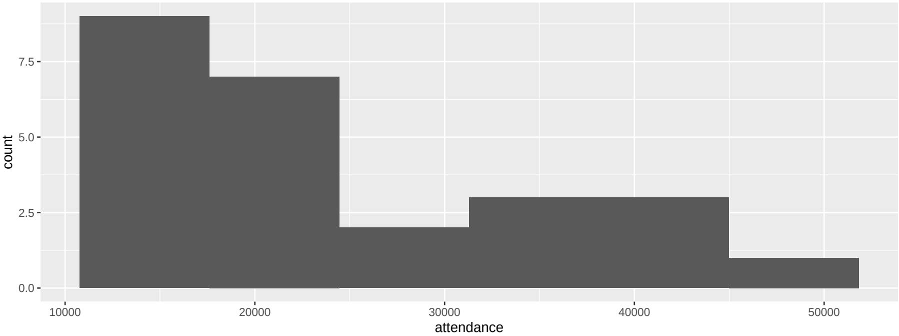
- This is a "big" data set: only 25 observations, but a lot of *variables*.
- To see the first few values in all the variables, can also use `glimpse`:

```
glimpse(jays)
```

```
Rows: 25
Columns: 21
$ row       <dbl> 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96~
$ game      <dbl> 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 27, 28, 29, 30, 31, 3~
$ date      <chr> "Monday, Apr 13", "Tuesday, Apr 14", "Wednesday, Apr 15", ~
$ box       <chr> "boxscore", "boxscore", "boxscore", "boxscore", "boxscore"~
$ team      <chr> "TOR", "TOR", "TOR", "TOR", "TOR", "TOR", "TOR", "TOR", "T~
$ venue     <lgl> NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA~
$ opp       <chr> "TBR", "TBR", "TBR", "TBR", "ATL", "ATL", "ATL", "BAL", "B~
$ result    <chr> "L", "L", "W", "L", "L", "W-wo", "L", "W", "W", "W", "W", ~
$ runs      <dbl> 1, 2, 12, 2, 7, 6, 2, 13, 4, 7, 3, 3, 5, 7, 7, 3, 10, 2, 3~
$ Oppruns   <dbl> 2, 3, 7, 4, 8, 5, 5, 6, 2, 6, 1, 6, 1, 0, 1, 6, 6, 3, 4, 4~
$ innings   <dbl> NA, NA, NA, NA, NA, 10, NA, NA, NA, NA, NA, NA, NA, NA, NA~
$ wl        <chr> "4-3", "4-4", "5-4", "5-5", "5-6", "6-6", "6-7", "7-7", "8~
$ position  <dbl> 2, 3, 2, 4, 4, 3, 4, 2, 2, 1, 4, 5, 3, 3, 3, 3, 5, 5, 5, 5~
$ gb        <chr> "1", "2", "1", "1.5", "2.5", "1.5", "1.5", "2", "1", "Tied~
$ winner    <chr> "Odorizzi", "Geltz", "Buehrle", "Archer", "Martin", "Cecil~
$ loser     <chr> "Dickey", "Castro", "Ramirez", "Sanchez", "Cecil", "Marimo~
$ save      <chr> "Boxberger", "Jepsen", NA, "Boxberger", "Grilli", NA, "Gri~
```

Attendance histogram

```
ggplot(jays, aes(x = attendance)) + geom_histogram(bins = 6)
```



Comments

- Attendances have substantial variability, ranging from just over 10,000 to around 50,000.
- Distribution somewhat skewed to right (but no outliers).
- These are a sample of “all possible games” (or maybe “all possible games played in April and May”). What can we say about mean attendance in all possible games based on this evidence?
- Think about:
  - Confidence interval
  - Hypothesis test.

Getting CI for mean attendance

- `t.test` function does CI and test. Look at CI first:

```
t.test(jays$attendance)
```

```

	One Sample t-test

data:  jays$attendance
t = 11.389, df = 24, p-value = 3.661e-11
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 20526.82 29613.50
sample estimates:
mean of x
 25070.16
```

- From 20,500 to 29,600.

Or, 90% CI

- by including a value for conf.level:

```
t.test(jays$attendance, conf.level = 0.90)
```

```

	One Sample t-test

data:  jays$attendance
t = 11.389, df = 24, p-value = 3.661e-11
alternative hypothesis: true mean is not equal to 0
90 percent confidence interval:
 21303.93 28836.39
sample estimates:
mean of x
 25070.16
```

- From 21,300 to 28,800. (Shorter, as it should be.)

Comments

- Need to say "column attendance within data frame `jays`" using $.
- 95% CI from about 20,000 to about 30,000.
- Not estimating mean attendance well at all!
- Generally want confidence interval to be shorter, which happens if:
  - SD smaller
  - sample size bigger
  - confidence level smaller
- Last one is a cheat, really, since reducing confidence level increases chance that interval won't contain pop. mean at all!

Another way to access data frame columns

```
with(jays, t.test(attendance))
```

```
	One Sample t-test

data:  attendance
t = 11.389, df = 24, p-value = 3.661e-11
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 20526.82 29613.50
sample estimates:
mean of x
 25070.16
```

Hypothesis test

- CI answers question "what is the mean?"
- Might have a value μ in mind for the mean, and question "Is the mean equal to μ, or not?"
- For example, 2014 average attendance was 29,327.
- "Is the mean this?" answered by **hypothesis test**.
- Value being assessed goes in **null hypothesis**: here, $H_0 : \mu = 29327$.
- **Alternative hypothesis** says how null might be wrong, eg. $H_a : \mu \neq 29327$.
- Assess evidence against null. If that evidence strong enough, *reject null hypothesis;* if not, *fail to reject null hypothesis* (sometimes *retain null*).
- Note asymmetry between null and alternative, and utter absence of word "accept".

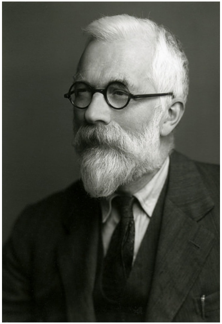
α and errors

- Hypothesis test ends with decision:
  - reject null hypothesis
  - do not reject null hypothesis.
- but decision may be wrong:

	Decision	
Truth	**Do not reject**	**reject null**
Null true	Correct	Type I error
Null false	Type II error	Correct

- Either type of error is bad, but for now focus on controlling Type I error: write $\alpha = \text{P(type I error)}$, and devise test so that α small, typically 0.05.
- That is, **if null hypothesis true**, have only small chance to reject it (which would be a mistake).
- Worry about type II errors later (when we consider power of test).

Why 0.05? This man.



- analysis of variance
- Fisher information
- Linear discriminant analysis
- Fisher's z-transformation
- Fisher-Yates shuffle
- Behrens-Fisher problem

Sir Ronald A. Fisher, 1890–1962.

Why 0.05? (2)

- From The Arrangement of Field Experiments (1926):

the line at about the level at which we can say: "Either there is something in the treatment, or a coincidence has occurred such as does not occur more than once in twenty trials." This level, which we may call the 5 per cent. point, would be indicated, though very roughly, by the greatest chance deviation observed in twenty successive trials. To

- and

If one in twenty does not seem high enough odds, we may, if we prefer it, draw the line at one in fifty (the 2 per cent. point), or one in a hundred (the 1 per cent. point). Personally, the writer prefers to set a low standard of significance at the 5 per cent. point, and ignore entirely all results which fail to reach this level. A scientific fact should be regarded as experimentally established only if a properly designed experiment *rarely fails* to give this level of significance. The very high

Three steps:

- from data to test statistic
  - how far are data from null hypothesis
- from test statistic to P-value
  - how likely are you to see "data like this" **if the null hypothesis is true**
- from P-value to decision
  - reject null hypothesis if P-value small enough, fail to reject it otherwise

Using t.test:

```
t.test(jays$attendance, mu=29327)
```

```

	One Sample t-test

data:  jays$attendance
t = -1.9338, df = 24, p-value = 0.06502
alternative hypothesis: true mean is not equal to 29327
95 percent confidence interval:
 20526.82 29613.50
sample estimates:
mean of x
 25070.16
```

- See test statistic -1.93, P-value 0.065.
- Do not reject null at $\alpha = 0.05$: no evidence that mean attendance has changed.

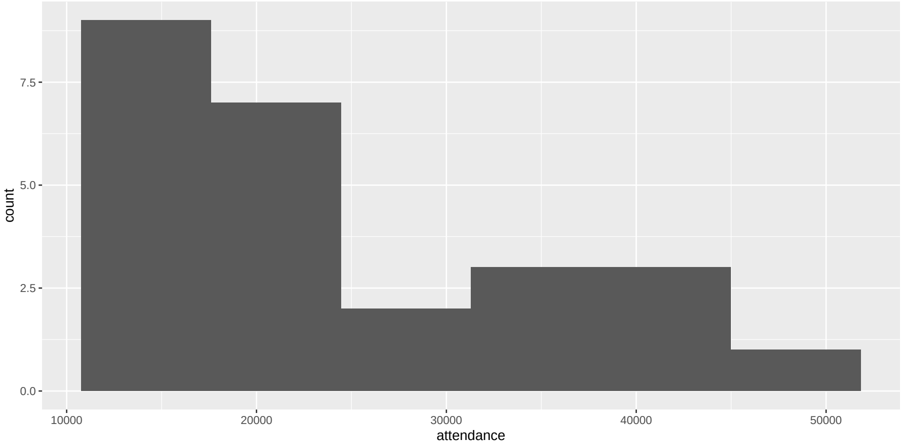
Assumptions

- Theory for t-test: assumes normally-distributed data.
- What actually matters is sampling distribution of sample mean: if this is approximately normal, t-test is OK, even if data distribution is not normal.
- Central limit theorem: if sample size large, sampling distribution approx. normal even if data distribution somewhat non-normal.
- So look at shape of data distribution, and make a call about whether it is normal enough, given the sample size.

Blue Jays attendances again:

- You might say that this is not normal enough for a sample size of $n = 25$, in which case you don't trust the t-test result:

```
ggplot(jays, aes(x = attendance)) + geom_histogram(bins = 6)
```



Another example: learning to read

- You devised new method for teaching children to read.
- Guess it will be more effective than current methods.
- To support this guess, collect data.
- Want to generalize to "all children in Canada".
- So take random sample of all children in Canada.
- Or, argue that sample you actually have is "typical" of all children in Canada.
- Randomization (1): whether or not a child in sample or not has nothing to do with anything else about that child.
- Randomization (2): randomly choose whether each child gets new reading method (t) or standard one (c).

Reading in data

- File at http://ritsokiguess.site/datafiles/drp.txt.
- Proper reading-in function is `read_delim` (check file to see)
- Read in thus:

```
my_url <- "http://ritsokiguess.site/datafiles/drp.txt"
kids <- read_delim(my_url," ")
```

The data

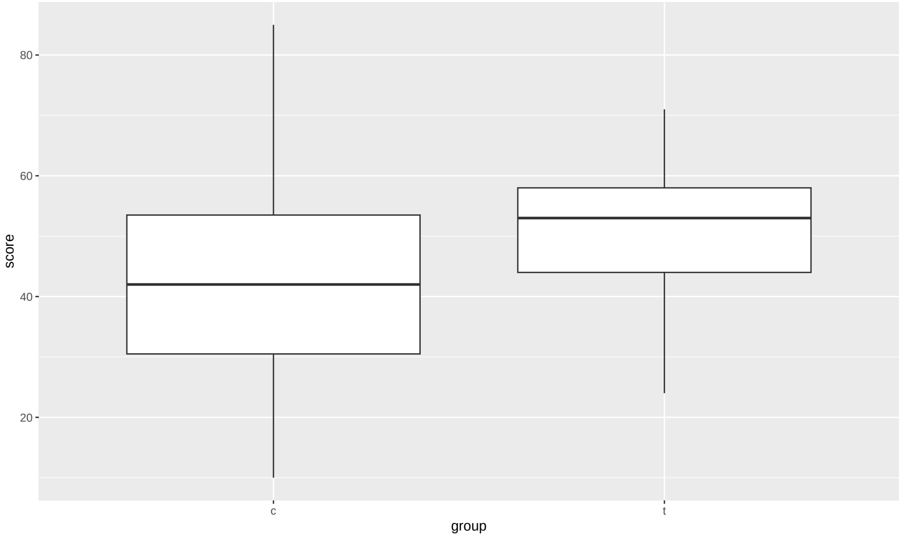
```
kids
```

```
# A tibble: 44 x 2
   group score
   <chr> <dbl>
 1 t        24
 2 t        61
 3 t        59
 4 t        46
 5 t        43
 6 t        44
 7 t        52
 8 t        43
 9 t        58
10 t        67
# i 34 more rows
```

In group, `t` is "treatment" (the new reading method) and `c` is "control" (the old one).

Boxplots

```
ggplot(kids, aes(x = group, y = score)) + geom_boxplot()
```



Two kinds of two-sample t-test

- pooled (derived in B57): $t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{(1/n_1) + (1/n_2)}}$,
  - where $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$
- Welch-Satterthwaite: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$
  - this t does not have exact t-distribution, but is approx t with non-integer df.

Two kinds of two-sample t-test

- Do the two groups have same spread (SD, variance)?
  - If yes (shaky assumption here), can use pooled t-test.
  - If not, use Welch-Satterthwaite t-test (safe).
- Pooled test derived in STAB57 (easier to derive).
- Welch-Satterthwaite is test used in STAB22 and is generally safe.
- Assess (approx) equality of spreads using boxplot.

The (Welch-Satterthwaite) t-test

- `c` (control) before `t` (treatment) alphabetically, so proper alternative is "less".
- R does Welch-Satterthwaite test by default
- Answer to "does the new reading program really help?"
- (in a moment) how to get R to do pooled test?

Welch-Satterthwaite

```
t.test(score ~ group, data = kids, alternative = "less")
```

```
    Welch Two Sample t-test

data:  score by group
t = -2.3109, df = 37.855, p-value = 0.01319
alternative hypothesis: true difference in means between grou
95 percent confidence interval:
      -Inf -2.691293
sample estimates:
mean in group c mean in group t
       41.52174        51.47619
```

The pooled t-test

```
t.test(score ~ group, data = kids,
       alternative = "less", var.equal = TRUE)
```

```

	Two Sample t-test

data:  score by group
t = -2.2666, df = 42, p-value = 0.01431
alternative hypothesis: true difference in means between group
95 percent confidence interval:
      -Inf -2.567497
sample estimates:
mean in group c mean in group t 
       41.52174        51.47619 
```

Two-sided test; CI

- To do 2-sided test, leave out `alternative`:

```
t.test(score ~ group, data = kids)
```

```

	Welch Two Sample t-test

data:  score by group
t = -2.3109, df = 37.855, p-value = 0.02638
alternative hypothesis: true difference in means between group
95 percent confidence interval:
 -18.67588  -1.23302
sample estimates:
mean in group c mean in group t 
       41.52174        51.47619 
```

Comments:

- P-values for pooled and Welch-Satterthwaite tests very similar (even though the pooled test seemed inferior): 0.013 vs. 0.014.
- Two-sided test also gives CI: new reading program increases average scores by somewhere between about 1 and 19 points.
- Confidence intervals inherently two-sided, so do 2-sided test to get them.

Jargon for testing

- Alternative hypothesis: what we are trying to prove (new reading program is effective).
- Null hypothesis: "there is no difference" (new reading program no better than current program). Must contain "equals".
- One-sided alternative: trying to prove better (as with reading program).
- Two-sided alternative: trying to prove different.
- Test statistic: something expressing difference between data and null (eg. difference in sample means, t statistic).
- P-value: probability of observing test statistic value as extreme or more extreme, if null is true.
- Decision: either reject null hypothesis or do not reject null hypothesis. **Never "accept"**.

Logic of testing

- Work out what would happen if null hypothesis were true.
- Compare to what actually did happen.
- If these are too far apart, conclude that null hypothesis is not true after all. (Be guided by P-value.)
- As applied to our reading programs:
  - If reading programs equally good, expect to see a difference in means close to 0.
  - Mean reading score was 10 higher for new program.
  - Difference of 10 was unusually big (P-value small from t-test). So conclude that new reading program is effective.
- Nothing here about what happens if null hypothesis is false. This is power and type II error probability.